SECURITY CLASSIFICATION OF THIS PAGE

AD-A245 005

REPORT DOCUMENTATION PAGE

DTIC ELECTE JAN 21 1992 S D

1a. REPORT SECURITY CLASSIFICATION	
1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
3. DISTRIBUTION / AVAILABILITY OF REPORT	Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	
5. MONITORING ORGANIZATION REPORT NUMBER(S)	ARO 22357.14-MA
6a. NAME OF PERFORMING ORGANIZATION	New York University
6b. OFFICE SYMBOL (If applicable)	
7a. NAME OF MONITORING ORGANIZATION	U. S. Army Research Office
6c. ADDRESS (City, State, and ZIP Code)	Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, NY 10012
7b. ADDRESS (City, State, and ZIP Code)	P. O. Box 12211, Research Triangle Park, NC 27709-2211
8a. NAME OF FUNDING / SPONSORING ORGANIZATION	U. S. Army Research Office
8b. OFFICE SYMBOL (If applicable)	
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	DAAL 03 86 K 0127
8c. ADDRESS (City, State, and ZIP Code)	P. O. Box 12211, Research Triangle Park, NC 27709-2211
10. SOURCE OF FUNDING NUMBERS	PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification)	Sojourns and Extremes of Stochastic Processes. Unclassified.
12. PERSONAL AUTHOR(S)	Simeon M. Berman
13a. TYPE OF REPORT	Final
13b. TIME COVERED	FROM 7/86 TO 7/89
14. DATE OF REPORT (Year, Month, Day)	1989-Sept. 21
15. PAGE COUNT	15
16. SUPPLEMENTARY NOTATION	The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Stochastic process, extreme value, sojourn time, local time, limiting distribution, self-intersections of paths, Gaussian process, Markov process.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Let $X(t)$, $t \in R^N$, be a stochastic process assuming values in R^M. The subjects of this investigation are three functionals associated with X: 1) The sojourn time of $X(t)$, $t \in B \subset R^N$, in a set $A \subset R^M$; it is denoted $L(A, B)$. 2) The local time of $X(t)$, which is the density of $L(A, B)$, as a function of sets A, with respect to Lebesgue measure. 3) The random variable $Z(B) = \sup(X(t) : t \in B)$ in the case $M = 1$. The object of the study is to describe specific distributional and structural properties of these functionals, and then formulate conditions on the finite-dimensional distributions of the process which are sufficient for these properties. It was shown how these conditions are related to the structures of classes of stochastic processes commonly arising in theoretical and applied probability.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT	☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS
21. ABSTRACT SECURITY CLASSIFICATION	Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL	Kathleen Kozar *Kathleen Kozar*
22b. TELEPHONE (Include Area Code)	212/998-3245
22c. OFFICE SYMBOL	

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

92-00840

92 1 17 025

S O J O U R N S AND E X T R E M E S OF S T O C H A S T I C P R O C E S S E S

FINAL REPORT

SIMEON M. BERMAN

SEPTEMBER 21, 1989

U. S. ARMY RESEARCH OFFICE

CONTRACT NUMBER DAAL 03 86 K 0127

NEW YORK UNIVERSITY

COURANT INSTITUTE OF MATHEMATICAL SCIENCES

NEW YORK, N.Y. 10012

Measure of the Multiple Self-Intersection Set of a Markov Process

SIMEON M. BERMAN
Courant Institute

Abstract

Let $X(t)$, $t \geqq 0$, be a Markov process in $\mathbf{R}^m$ with homogeneous transition density $p(t; x, y)$. For a closed bounded set $B \subset R^m$, X is said to have a self-intersection of order $r \geqq 2$ in B if there are distinct points $t_1 < \cdots < t_r$ such that $X(t_1) \in B$ and $X(t_j) = X(t_1)$, for $j = 2, \cdots, r$. The focus of this work is the Hausdorff measure, suitably defined, of the set of such r-tuples. The main result is that under general conditions on $p(t; x, y)$ as well as the specific condition

$$\int_0^t \sup_{x \in B} \left(\int_B p^r(s; x, y)\, dy \right)^{1/r} ds < \infty,$$

there is a measure function $M(t)$, defined explicitly in terms of the integral above, such that the corresponding Hausdorff measure of the self-intersection set is positive, with positive probability. The results are applied to Lévy and diffusion processes, and are shown to extend recent results in this area.

1. Introduction and Summary

Let $X(t)$, $t \geqq 0$, be a homogeneous Markov process in $\mathbf{R}^m$, $m \geqq 1$, having the transition density function $p(t; x, y)$ representing the density of $X(t)$ at y, conditioned by $X(0) = x$. The focus of this paper is the magnitude of the set of points in the domain at which the sample function intersects itself at least r times, for fixed $r \geqq 1$. Many authors have considered the following formulation. Let $I_1, \cdots, I_r$ be closed bounded disjoint time intervals, and define the subset of the product set $I_1 \times \cdots \times I_r$,

$$S(I_1, \cdots, I_r) = \left\{ (t_1, \cdots, t_r) : t_j \in I_j,\ X(t_1) = \cdots = X(t_r) \right\}.$$

Some of the well-known results in this area provide the Hausdorff dimensions of this set for particular kinds of processes, and the current paper provides a contribution in this direction.

Let us recall the elements of Hausdorff measure. Let C be a compact metric space, and let $\mathscr{C}$ be a class of subsets J whose union contains C. For a

Communications on Pure and Applied Mathematics, Vol. XLIII 575–598 (1990)
 CCC 0010-3640/90/050575-24$04.00

Self-intersections and Local Nondeterminism of Gaussian Processes

Simeon M. Berman

Courant Institute of Mathematical Sciences

New York University

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant DMS 88 01188 and the U. S. Army Research Office, Contract DAAL 03 86 K 0127.

AMS Subject Classification (1985): 60G15, 60G17, 60J55

Key words and phrases: Intersections of sample paths, local nondeterminism, local time, Gaussian processes, spectral distribution.

Running head: Self-intersections of Gaussian processes

Lecture Notes in Statistics 51, Extreme Value Theory, Proceedings, Oberwolfach 1987, Springer-Verlag 1989, pp. 81-99.

A CENTRAL LIMIT THEOREM FOR EXTREME SOJOURN TIMES OF STATIONARY GAUSSIAN PROCESSES

Simeon M. Berman[1]
Courant Institute of Mathematical Sciences, New York University
251 Mercer Street, New York 10012

Abstract

Let $X(t)$, $t \geq 0$, be a real measurable stationary Gaussian process with mean 0 and covariance function $r(t)$. For a given measurable function $u(t)$ such that $u(t) \to \infty$ for $t \to \infty$, let L_t be the sojourn time of $X(s)$, $0 \leq s \leq t$, above $u(t)$. Assume that the spectral distribution function in the representation of $r(t)$ is absolutely continuous; then $r(t)$ also has the representation $r(t) = \int b(t+s)b(s)ds$, where $b \in L_2$. The main result is: If $b \in L_1$, and if $u(t)$ increases sufficiently slowly, then $(L_t - EL_t)/(\mathrm{Var}(L_t))^{1/2}$ has a limiting standard normal distribution for $t \to \infty$. The allowable rate of increase of $u(t)$ with t is specified.

1. Introduction and Summary.

Let $X(t)$, $t \geq 0$, be a real measurable stationary Gaussian process with mean 0 and covariance function $r(t) = EX(0)X(t)$. For simplicity we take $r(0) = 1$. For $t > 0$, let $L_t(u)$ be the sojourn time of $X(s)$, $0 \leq s \leq t$, above the level $u : L_t(u) = mes(s : 0 \leq s \leq t, X(s) > u)$. Then for a given measurable function $u(t)$, we define

$$L_t = L_t(u(t)) = \int_0^t 1_{[X(s) > u(t)]}\, ds. \tag{1.1}$$

The main result of this paper is a new limit theorem for the distribution of L_t, for $t \to \infty$, where $u(t)$ increases at a specified rate with t. We assume that the spectral distribution function in the representation of $r(t)$ is absolutely continuous. Then $r(t)$ also has the representation (see, for example, [7], page 532),

$$r(t) = \int_{-\infty}^{\infty} b(t+s)b(s)ds, \tag{1.2}$$

where $b(s)$ is the Fourier transform of the square root of the spectral density, and

$$\int_{-\infty}^{\infty} |b(s)|^2 ds < \infty. \tag{1.3}$$

Since $X(t)$ is real valued, $b(s)$ is also real valued. Our main result is:

[1] This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant DMS 85 01512, and the U. S. Army Research Office, Contract DAAL 03 86 K 0127.

LOCAL TIME OF A FUNCTION OF A STOCHASTIC PROCESS

By

Simeon M. Berman
Courant Institute of Mathematical Sciences
New York University
251 Mercer Street. New York 10012.

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant DMS 85 01512, and the Army Research Office, Contract DAAL 03 86 K 0127.

AMS Classification 1985: 60G17, 60J55.

Key words and phrases: Local time, sample function, function of a stochastic process, mth power integrability of local time.

Running head: Local time of function.

Adv. Appl. Prob. 19, 106–122 (1987)
Printed in N. Ireland

POISSON AND EXTREME VALUE LIMIT THEOREMS FOR MARKOV RANDOM FIELDS

SIMEON M. BERMAN,* *Courant Institute of Mathematical Sciences*

Abstract

Let X_t, $t \in Z^m$, be a Markov random field assuming values in R^M. Let I_n be a rectangular box in Z^m with its center at 0 and corner points with coordinates $\pm n$. Let (A_n) be a sequence of measurable subsets of R^M such that $P(X_t \in A_n \mid X_s,\ s \in \text{neighborhood of } t) \to 0$, for $n \to \infty$; and let $f_n(x)$ be the indicator of A_n. Under appropriate conditions on the nearest-neighbor distributions of (X_t), the conditional distribution of $\sum_{t \in I_n} f_n(X_t)$, given the values of X_s, for s on the boundary of I_n, converges to the Poisson distribution. An immediate application is an extreme value limit theorem for a real-valued Markov random field.

STATIONARITY; MIXING; EXTREME VALUES; POISSON LIMIT

1. Introduction and summary

The main result of this paper is a limit theorem for the extreme values of a Markov random field on a discrete lattice. Let X_t, $t \in T$, be a family of real random variables on some probability space, and where T is a countable index set. Let I_n, $n \geqq 1$, be a sequence of finite subsets of T such that $I_n \subset I_{n+1}$ and $T = \bigcup_{n \geqq 1} I_n$. Define

$$M_n = \max_{t \in I_n} X_t. \tag{1.1}$$

The extreme value limit problem is as follows: we seek conditions under which there exist a non-degenerate distribution function $G(x)$ and real sequences (a_n) and (b_n), with $a_n > 0$ such that

$$\lim_{n \to \infty} P(a_n^{-1}(M_n - b_n) \leqq x) = G(x) \tag{1.2}$$

at all continuity points x. When the X_t's are independent with a common

Received 5 September 1985, revision received 21 January 1986.

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the U.S. Army Research Office, Grant number DAAG-29-85-K-0146, and the National Science Foundation, Grant DMS 85 01512.

* Postal address: Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012, USA.

SIAM J. APPL. MATH.
Vol. 49, No. 2, pp. 608-616, April 1989

015

SOJOURN TIMES IN A CONE FOR A CLASS OF VECTOR GAUSSIAN PROCESSES*

SIMEON M. BERMAN†

Abstract. Let $\mathbf{X}(t)$ be a Gaussian process in R^m with stationary increments. Let $\mathbf{S}(t)$ be the covariance matrix of $\mathbf{X}(t)-\mathbf{X}(0)$, and assume that there is a positive real function $\sigma^2(t)$ and a positive definite matrix $\mathbf{R}$ such that $\mathbf{S}(t)\sim\sigma^2(t)\mathbf{R}$, for $t\to 0$. Let $f(\mathbf{x})$, $\mathbf{x}\in R^m$, be a real bounded Borel function such that $f(c\mathbf{x})=f(\mathbf{x})$, for all $c>0$. The main result is that $t^{-1}\int_0^t f(\mathbf{X}(s)-\mathbf{X}(0))\,ds$ has, for $t\to 0$, a limiting distribution, which is identified. A particular case of this is a new result for the limiting uniform distribution of the positive sojourn time proportion for $m=1$. Another application is a limit theorem for the conditional distribution of the sojourn time proportion above a high level.

Key words. Gaussian vector process, stationary increments, sojourn time, local time, high level, cone, slow variation, tightness, weak convergence

AMS(MOS) subject classifications. 60G15, 60G10

1. Introduction and summary. Let K be a cone in R^m, that is, a Borel set such that $\mathbf{x}\in K$ implies $c\mathbf{x}\in K$ for every $c>0$. Let $\mathbf{X}(t)$, $t\geqq 0$, be a measurable stochastic process assuming values in R^m. The main result of this paper is concerned with the limiting distribution of the proportion of time spent in K by $\mathbf{X}(s)-\mathbf{X}(0)$, $0\leqq s\leqq t$, for $t\to 0$, in the case where $\mathbf{X}(\cdot)$ belongs to a particular class of Gaussian processes.

Let $\mathbf{X}(t)$, $t\geqq 0$, be a measurable Gaussian process in R^m with mean $\mathbf{0}$ and stationary increments. For any vector $\mathbf{x}$, the notation $\mathbf{x}'$ will be used for the transpose. Put

$$\mathbf{S}(t)=E[\mathbf{X}(t)-\mathbf{X}(0)][\mathbf{X}(t)-\mathbf{X}(0)]'. \tag{1.1}$$

Assume that $\mathbf{S}(t)$ is continuous for $t\geqq 0$, positive definite for $t>0$, and $\mathbf{S}(0)=\mathbf{0}$-matrix. Our main assumption about $\mathbf{S}$ is the following hypothesis.

Hypothesis. There is a positive definite matrix $\mathbf{R}$ and a continuous positive function $\sigma^2(t)$, $t>0$, which is slowly varying for $t\to 0$, such that

$$\mathbf{S}(t)/\sigma^2(t)\to\mathbf{R}\quad\text{for } t\to 0. \tag{1.2}$$

Here $\mathbf{S}(t)/\sigma^2(t)$ is the matrix $\mathbf{S}$ times the scalar $1/\sigma^2$. Recall that a function σ^2 is slowly varying if $\sigma^2(tx)/\sigma^2(t)\to 1$, for $t\to 0$, for every $x>0$.

Let $f(\mathbf{x})$, $\mathbf{x}\in R^m$, be a real bounded Borel function. It is called a *cone function* if

$$f(c\mathbf{x})=f(\mathbf{x})\quad\text{for all } c>0,\ \mathbf{x}\in R^m. \tag{1.3}$$

The indicator of a cone in R^m is obviously a cone function.

While the main theorem is of interest for indicators of cones it is stated more generally for cone functions.

THEOREM 1. *Let $\mathbf{Z}$ be a normal random vector with mean $\mathbf{0}$ and covariance matrix $\mathbf{R}$, and let $\phi_{\mathbf{R}}(z)$, $z\in R^m$, be the corresponding normal density. Under the conditions on $\mathbf{X}(t)$ and $\mathbf{S}(t)$ stated above, if f is a cone function, then*

$$t^{-1}\int_0^t f(\mathbf{X}(s)-\mathbf{X}(0))\,ds \tag{1.4}$$

* Received by the editors July 31, 1987; accepted for publication (in revised form) March 22, 1988.

† Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York, 10012. This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, grant DMS-85-01512, and the Army Research Office, contract DAAL-03-86-K-0127.

The Modulator of the Local Time

SIMEON M. BERMAN
Courant Institute

Abstract

Let $x(t)$, $0 \leqq t \leqq 1$, be a real measurable function having a local time $\alpha(x, t)$ which is a continuous function of t for almost all x. It is also assumed that, for some $m \geqq 2$ and some real interval B, $\alpha^m(x, 1)$ is integrable over B. The modulator is a function $M_m(t, B)$, $t > 0$, defined in terms of α. It is shown that the modulator serves as a measure of the smoothness of the $L_m(B)$-valued function $\alpha(\cdot, t)$ with respect to t. Then it is shown that the modulator plays a central role in precisely describing certain irregularity properties of $x(t)$. The results are applied to the case where $x(t)$ is the sample function of a real stochastic process. In this way new results are obtained for large classes of Gaussian and Markov processes.

1. Introduction and Summary

Let $x(t)$, $0 \leqq t \leqq 1$, be a real measurable function. For every pair of linear Borel sets A and I, $I \subset [0,1]$, define $\nu(A, I) =$ Lebesgue measure $\{t\colon t \in I, x(t) \in A\}$. If, for fixed I, $\nu(\cdot, I)$ is absolutely continuous as a measure of sets A, then its Radon-Nikodym derivative, which we denote by $\alpha_I(x)$, is called the local time of $x(t)$, $t \in I$. It satisfies

$$\nu(A, I) = \int_A \alpha_I(x)\, dx. \tag{1.1}$$

Define

$$\alpha(x, t) = \alpha_{[0,t]}(x). \tag{1.2}$$

In [3] and [6] we proved that the function $x(t)$ has specified irregularity properties under the hypothesis of "temporal continuity" of the local time, which means that

(1.3) $\quad \alpha(x, t)$ *is continuous in* t *for almost all* x.

In the present paper we prove additional irregularity properties of $x(t)$ under the additional assumption

$$\int_B \alpha^m(x, 1)\, dx < \infty \tag{1.4}$$

for some real interval B and some integer $m \geqq 2$. The assumption (1.4) implies

$$\int_B \alpha^m(x, t)\, dx < \infty$$

for all $0 \leqq t \leqq 1$ because $\alpha(x, t)$ may be assumed to be monotonic in t (see [1]).

Communications on Pure and Applied Mathematics, Vol. XLI 121–132 (1988)
 CCC 0010-3640/88/010121-12$04.00

COMMUN. STATIST.-STOCHASTIC MODELS, 4(1), 1-43 (1988)

Sojourns and Extremes of a Stochastic Process Defined as a Random Linear Combination of Arbitrary Functions

Simeon M. Berman

Courant Institute of Mathematical Sciences
251 Mercer Street
New York, New York 10012.

ABSTRACT

Let X_t be a real stochastic process of the form $(\mathbf{X},\mathbf{f}(t))$, where $\mathbf{X}$ is a random vector in R^n with an orthogonally invariant distribution, and $\mathbf{f}(t), 0 \le t \le 1$, assumes values in R^n. Put $x_0 = \sup(x: P(\|\mathbf{X}\| \le x) < 1)$, finite or infinite. For real $u < x_0$, let L_u be the sojourn time of X_t, $0 \le t \le 1$, above u. The main results are limit theorems for the distribution of L_u and for $P(\max_t X_t > u)$, for $u \to x_0$. The hypotheses are stated in terms of the conditions on the tail of the distribution of $\mathbf{X}$ which are used in extreme value theory for i.i.d. random variables.

1. INTRODUCTION AND SUMMARY

The subject of this paper is the study of certain sojourn time and extreme value limit theorems for a particular class of real stochastic processes. Let $\mathbf{X} = (X_1,\dots,X_n)$ be real random vector in R^n and $\mathbf{f}(t) = (f_1(t),\dots,f_n(t)), 0 \le t \le 1$, a real vector valued function. Then,

$$X_t=(\mathbf{X},\mathbf{f}(t))=\sum_{j=1}^{n} X_j f_j(t), \quad 0 \le t \le 1. \tag{1.1}$$

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant DMS 85 01512, and the Army Research Office, Contract DAAL 03 86 K 0127.

AMS Subject Classification 1985 Primary 60G17, Secondary 60K99.

Key words and phrases Sojourns of a process, extremes of a process, distribution of norm, orthogonal invariance, domain of attraction, extreme value distribution.

THE MAXIMUM OF A GAUSSIAN PROCESS WITH NONCONSTANT VARIANCE: A SHARP BOUND FOR THE DISTRIBUTION TAIL

by

SIMEON M. BERMAN and NORIO KÔNO

Reprin'ed From: The Annals of Probability
Volume 17 Number 2 April 1989

Local Times of Stochastic Processes with Applications to the Sample Functions of Markov Processes

by

Simeon M. Berman

Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York 10012

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant DMS 85 01512 , and the Army Research Office, Contract DAAL 03-86-K-0127.

AMS 1985 Subject Classification: Primary: 60J55, 60G17, Secondary: 60J25.

Key words and phrases: Local time, sample function, Markov process, approximate local growth, measure of level set, γ-measure variation

Running head: Local Times of Stochastic Processes

Stochastic Processes and their Applications 23 (1986) 281-290
North-Holland

THE SUPREMUM OF A PROCESS WITH STATIONARY INDEPENDENT AND SYMMETRIC INCREMENTS

Simeon M. BERMAN

Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York 10012, USA

Received 14 April 1986
Revised 14 August 1986

Let X_t, $t \geq 0$, be a process with stationary independent and symmetric increments. If the tail of the Lévy spectral measure in the representation of the characteristic function is of regular variation of index $-\alpha$, for some $0<\alpha<2$, then $P(\sup(X_s: 0 \leq s \leq t) > u) \sim P(X_t > u)$, for $u \to \infty$, for each $t > 0$.

AMS (1985) *Subject Classification*: 60J30, 60F10.

Independent increments * supremum distribution * regular variation * sojourn above high level

1. Introduction and summary

Let X_t, $t \geq 0$, be a separable stochastic process with independent increments, which is centered and has no fixed points of discontinuity. Then there is a version of the process which, with probability 1, has sample functions which are locally bounded and for which $\sup(X_s: 0 \leq s \leq t)$ is a well defined random variable. The main result of this paper is the asymptotic relation

$$P(\sup(X_s: 0 \leq s \leq t) > u) \sim P(X_t > u) \quad \text{for } u \to \infty, \tag{1.1}$$

for a large class of such processes.

Suppose that the increments are stationary. Then, by the classical result of Lévy, the characteristic function of $X_t - X_s$ for $0 \leq s < t$ is of the form

$$E\, e^{i\theta(X_t - X_s)} = e^{-(t-s)f(\theta)},$$

where $e^{-f(\theta)}$ is the characteristic function of an infinitely divisible distribution. In the particular case which we will consider here, where the increments are symmetrically distributed about 0, the function $f(\theta)$ takes the form

$$f(\theta) = 2\int_0^\infty (1-\cos\theta x)\frac{1+x^2}{x^2}\,dG(x), \tag{1.2}$$

This paper has been sponsored by the National Science Foundation, Grant DMS 85 01512, and the Army Research Office, Contract DAAL 03-86-K-0127.

SPECTRAL CONDITIONS FOR LOCAL NONDETERMINISM

Simeon M. BERMAN
Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, 10012, USA

Received 13 July 1987
Revised 3 August 1987

Let $X(t)$ be a real Gaussian process with stationary increments and spectral distribution function $F(x)$. Put $\phi(t) = F(\infty) - F(1/t)$. Sufficient conditions in terms of F are given for the process to be locally ϕ-nondeterministic. These are formulated for discrete and absolutely continuous functions F. The results in the discrete case are applied to the analysis of the local time of a random Fourier series with i.i.d. coefficients. The class of distributions of the coefficients includes not only the normal distribution but others such as the symmetric stable distribution.

AMS (1985) *Subject Classifications*: 60G10, 60G15, 60J55.

local nondeterminism • local time • Gaussian process • stationarity • spectral distribution • random Fourier series

1. Introduction and summary

Let $X(t)$, $t \geq 0$, be a separable Gaussian process with mean 0, and let J be an open interval on the t-axis. Assume that there exists $d > 0$ such that

$$\begin{aligned} &EX^2(t) > 0, \quad t \in J, \quad \text{and} \\ &E(X(t) - X(s))^2 > 0 \quad \text{for} \quad 0 < |t-s| < d, \quad s, t \in J. \end{aligned} \tag{1.1}$$

The concept of local nondeterminism (LND) was introduced by the author in [2]. According to Lemmas 2.1 and 2.2 of that paper, the definition of LND is equivalent to the following: For every $m \geq 2$, let $t_1 < t_2 < \cdots < t_m$ be variable ordered points in J; then the determinant of the covariance matrix of the m standardized random variables,

$$\frac{X(t_1)}{(\operatorname{Var} X(t_1))^{1/2}}, \frac{X(t_2) - X(t_1)}{(\operatorname{Var}(X(t_2) - X(t_1)))^{1/2}}, \ldots, \frac{X(t_m) - X(t_{m-1})}{(\operatorname{Var}(X(t_m) - X(t_{m-1})))^{1/2}}$$

is, as a function of $t_1 < \cdots < t_m$, bounded away from 0. The concept was extended by Cuzick [4], who defined local ϕ-nondeterminism, LND (ϕ), by replacing the

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University under the sponsorship of the National Science Foundation, Grant DMS 85 01512, and the Army Research Office, Contract DAAL 03-86-K-0127.

0304-4149/88/$3.50 © 1988, Elsevier Science Publishers B. V. (North-Holland)

Ann Prob

EXTREME SOJOURNS OF DIFFUSION PROCESSES

by

Simeon M. Berman
Courant Institute of Mathematical Sciences
New York University
251 Mercer Street, New York 10012

This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the National Science Foundation, Grant DMS 85 01512, and the Army Research Office, Contract No. DAAL 03-86-K-0127.

AMS Subject Classification (1985): Primary 60J60, Secondary 60G17

Key Words and phrases: Diffusion process, sojourn time, first passage time, local time , scale function.

Short title: Sojourns of diffusion